A Principle of Pattern Formation and Recognition

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Abstract – The paper proposes a principle of pattern formation and recognition, which meets the requirements of self-contained adaptive control.

INTRODUCTION

As will be recalled, pattern recognition theory [1, 2, 3] presupposes the use of *a priori* information in the form of a learning set, a classification principle, a class alphabet, and a dictionary of features, among other things. It cannot be doubted, however, that this *a priori* information must, in the general case, be generated as part of a unified process interrelated with recognition. This stems from the general statement of the problem of object control, where the object interacts uniquely and independently with an environment whose properties are little known *a priori*, if at all.

This problem has of late been growing ever more important in practical applications of microprocessor control (in medical equipment, robots, deep-water probes and space probes, etc.), in data-processing systems, in artificial intelligence systems, in the analysis of biological control systems, and in cognition theory.

The general cybernetic issues bearing on the present subject are examined in Ref. [4]. However, the control theory which would fit in with the above statement of the problem now lies outside the author's field of view, except perhaps the control concept set forth in Refs. [5 - 7]. This concept maintains, among other things, that the unique interaction of the controlled object (CO) with the environment makes it necessary to match the pattern formation and recognition (PFR) principle to the action generation and decision-making process.

This paper will attempt to expound the PFR principle proper, matched to the above control concept, while divorcing it from the decision-making process, wherever possible.

1. STATEMENT OF THE PFR PROBLEM WITHIN THE FRAMEWORK OF THE NATURAL CONTROL CONCEPT

The statement of the PFR problem in a system satisfying the principles of natural control [7] stems from the formal method of system description [5] and from the principle underlying the organization of control systems [6, 7].

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1.1. Formalization of System Objects

By *macroobjects* we mean the entities which form the system covering, such as the environment, the controlled object (CO), and the control system (CS), and their unions, intersections, and complements. We will represent macroobjects by directed, weighed, disconnected graphs and subgraphs and define them by named pairs which denote the set of *element*-representing vertices and the set of *effect*-representing connections [5]. Formally, a macroobject is identical to a system element. An element with zero output arity is a *sink*, and an element with zero input arity is a *source*. The law by which a source operates is not defined. The various measures of an action will be called its degrees of freedom (DFs).

We describe each macroobject in the following order: (I) the name and designation of a graph, (II) input connections of the graph, (III) the elements of the object incidental to the latter, (IV) DFs of input effects, (V) DFs of the input effects sensed by the object, (VI) output connections of the object, (VII) the elements of the object incidental to the latter, (VIII) DFs of output effects, and (IX) DFs of the output effects initiated by the object.

1.1.1. (I) *The system* or *environment in the strict sense* $\mathfrak{U}(V, E)$; (II) - (IX) is not defined.

1.1.2. (I) *The environment in the broad sense* $\mathfrak{S}(V', E') \subset \mathfrak{U}$; $V' \subset V$; $E' \subset E$. (II) $\mathbf{Y}_K(Y_1, Y_2, \ldots, Y_j, \ldots, Y_K)$; $|\mathbf{Y}_K| = K$. (III) $\tilde{Y}_K \subseteq V'$. (IV) $\mathcal{Y}_K(\{\mathfrak{y}_1\}, \{\mathfrak{y}_2\}, \ldots, \{\mathfrak{y}_j\}, \ldots, \{\mathfrak{y}_K\})$. (V) $\{\mathfrak{y}_j^{\Gamma}\} \subseteq \{\mathfrak{y}_j\}$; $\mathfrak{y}_{j,\gamma}^{\Gamma} \in \{\mathfrak{y}_j^{\Gamma}\}$; $v_\gamma = \{\mathfrak{y}_{j,\gamma}^{\Gamma}\}$, where $\gamma \in (1, 2, \ldots, G)$ and $\Gamma = \{v_\gamma\}$. (VI) $\mathbf{X}_M(X_1, X_2, \ldots, X_i, \ldots, X_M)$; $|\mathbf{X}_M| = M$. (VII) $\tilde{X}_M \subseteq V'$. (VIII) $\mathcal{X}(\{\mathfrak{x}_1\}, \{\mathfrak{x}_2\}, \ldots, \{\mathfrak{x}_i\}, \ldots, \{\mathfrak{x}_M\})$. (IX) $\{\mathfrak{x}_i^{\Psi}\} \subseteq \{\mathfrak{x}_i\}$; $\mathfrak{x}_{i,\varepsilon}^{\Psi} \in \{\mathfrak{x}_i^{\Psi}\}$; $\psi_\varepsilon = \{\mathfrak{x}_{i,\varepsilon}^{\Psi}\}$, where $\varepsilon \in (1, 2, \ldots, E)$ and $\Psi = \{\psi_\varepsilon\}$.

1.1.3. (I) *The controlled object* $\mathfrak{R}(D, F) \subset \mathfrak{U}$; $D \subset V$; $F \subset E$; $D \cap V' = \Lambda$ (empty set), $F \cap E' = \Lambda$; $D \cup V' = V$; $F \cup E' \subset E$. (II) $\mathbf{X}_M$. (III) $\tilde{X}''_M \subseteq D$. (IV) $\mathcal{X}$. (V) $\{\mathfrak{x}_i^{X}\}$; $\{\mathfrak{x}_i^{\Psi}\} \cup \{\mathfrak{x}_i^{X}\} \subseteq \{\mathfrak{x}_i\}$; $\{\mathfrak{x}_{i,\iota}^{X}\} \in \{\mathfrak{x}_i^{X}\}$; $\chi_\iota = \{\mathfrak{x}_{i,\iota}^{X}\}$, where $\iota \in (1, 2, \ldots, M)$ and $X = \{\chi_\iota\}$.

(VI) $\mathbf{Y}_K$. (VII) $\tilde{Y}''_K \subseteq D$. (VIII) $\mathcal{Y}_K$. (IX) $\{\mathfrak{y}_j^{\Upsilon}\}$; $\{\mathfrak{y}_j^{\Gamma}\} \cup \{\mathfrak{y}_j^{\Upsilon}\} \subseteq \{\mathfrak{y}_j\}$; $\mathfrak{y}^{\Upsilon}_{j,\gamma} \in \{\mathfrak{y}_j^{\Upsilon}\}$; $\upsilon_j = \{\mathfrak{y}^{\Upsilon}_{j,j}\}$, where $j \in (1, 2, \ldots, K)$ and $\Upsilon = \{\upsilon_j\}$.

1.1.4. (I) *The environment in the narrow sense* $\mathfrak{W}(V'', E'') \subset \mathfrak{U}$; $V'' \subset V$; $E'' \subset E$. (II) $\mathbf{Y}_L(y_1, y_2, \ldots, y_\lambda, \ldots, y_L)$; $|\mathbf{Y}_L| = L$. (III) $\tilde{Y}''_L \subseteq V''$. (IV) $\mathcal{F}(\{\varphi_1\}, \{\varphi_2\}, \ldots, \{\varphi_\lambda\}, \ldots, \{\varphi_L\})$. (V) $\{\varphi_\lambda^{Z}\} \subseteq \{\varphi_\lambda\}$; $\varphi^{Z}_{\lambda,\zeta} \in \{\varphi^{Z}_\lambda\}$; $\delta_\zeta = \{\varphi^{Z}_{\lambda,\zeta}\}$, where $\zeta \in (1, 2, \ldots, z)$ and $\mathbf{Z} = \{\delta_\zeta\}$. (VI) $\mathbf{X}_{\mathrm{P}}(x_1, x_2, \ldots, x_z, \ldots, x_{\mathrm{P}})$; $|\mathbf{X}_{\mathrm{P}}| = \mathrm{P}$. (VII) $\tilde{X}''_{\mathrm{P}} \subseteq V''$. (VIII) $\mathcal{K}(\{\kappa_1\}, \{\kappa_2\}, \ldots, \{\kappa_z\}, \ldots, \{\kappa_{\mathrm{P}}\})$. (IX) $\{\kappa_z^{\Sigma}\} \subseteq \{\kappa_z\}$; $\kappa^{\Sigma}_{z,\varsigma} \in \{\kappa^{\Sigma}_z\}$; $\mu_\varsigma = \{\kappa^{\Sigma}_{z,\varsigma}\}$, where $\varsigma \in (1, 2, \ldots, R)$ and $\Sigma = \{\mu_\varsigma\}$.

1.1.5. (I) *The control system* $\mathfrak{B}(W, C) \subset \mathfrak{R}$; $W \subset D$; $C \subset F$, W and C are of informative nature. (II) $\mathbf{X}_{\mathrm{P}}$, where $x_z \in \mathbf{X}_{\mathrm{P}}$ is an *input pole*. (III) $\tilde{X}'_{\mathrm{P}} \subseteq W$. (IV) $\mathcal{K}$. (V) $\{\kappa_z^{\Xi}\}$; $\{\kappa_z^{\Sigma}\} \cup \{\kappa_z^{\Xi}\} \subseteq \{\kappa_z\}$; $\kappa^{\Xi}_{z,\xi} \in \{\kappa^{\Xi}_z\}$; $\nu_\xi = \{\kappa^{\Xi}_{z,\xi}\}$, where $\xi \in (1, 2, \ldots, N)$ and $\Xi = \{\nu_\xi\}$. (VI) $\mathbf{Y}_L$, where $y_\lambda \in \mathbf{Y}_L$ is an *output pole*. (VII) $\tilde{X}'_L \subseteq W$. (VIII) $\mathcal{F}$. (IX) $\{\varphi_\lambda^{\Omega}\}$; $\{\varphi_\lambda^{\Omega}\} \cup \{\varphi_\lambda^{Z}\} \subseteq \{\varphi_\lambda\}$; $\varphi^{\Omega}_{\lambda,\eta} \in \{\varphi^{\Omega}_\lambda\}$; $\vartheta_\eta = \{\varphi^{\Omega}_{\lambda,\eta}\}$, where $\eta \in (1, 2, \ldots, Q)$ and $\Omega = \{\vartheta_\eta\}$.

1.1.6. (I) *The sensor unit* (SU) $\mathfrak{D} \subset \mathfrak{R}$ (II) through (V) correspond to (II) through (V) in $\mathfrak{R}$ (VI) through (IX) correspond to (VI) through (IX) i the environment $\mathfrak{W}$.

1.1.7. (I) *An actuator* (Act.) $\mathfrak{E} \subset \mathfrak{R}$. (II) through (V) correspond to (II) through (V) in the environment $\mathfrak{W}$. (VI) through (IX) correspond to (VI) through (IX) in $\mathfrak{R}$.

The laws by which the objects $\mathfrak{S}$, $\mathfrak{R}$, $\mathfrak{W}$, $\mathfrak{B}$, $\mathfrak{D}$, and $\mathfrak{E}$ operate are defined, respectively, by the relations: $\sigma : \Gamma \to \Psi$ and $s : \Upsilon \to \mathrm{X}$; $o : \mathrm{X} \to \Upsilon$; $w : \mathrm{Z} \to \Sigma$; $b : \Xi \to \Omega$ and $c : \Sigma \to \mathrm{Z}$; $d : \mathrm{X} \to \Sigma$; $e : \mathrm{Z} \to \Upsilon$.

We define the following relations: $a_1 : \Upsilon \to \Gamma$; $a_2 : \Psi \to \mathrm{X}$; $a_3 : \Sigma \to \Xi$; $a_4 : \Omega \to \mathrm{Z}$, and $\beta : \{\kappa_z^{\Sigma}\} \to \{\kappa_z^{\Xi}\}$. We will also use other notation, such as $\chi_\iota = \upsilon_j s$ or $\upsilon_j s \chi_\iota$, where $\upsilon_j \in \Upsilon$ and $\chi_\iota \in \mathrm{X}$. Time $T = \{t\}$ is deemed discrete; t_1 marks the instant when the CS begins operating, t_{p} is the present (or current) instant, and t_{f} marks the final instant or the instant when the CS ceases operating. Further, $t_1, t_{\mathrm{p}}, t_{\mathrm{f}} \in T$.

1.2. The Controlled Interaction

We call the sequence of routes in $\mathfrak{U}$ which begins at a source and ends at a sink *a macroprocess*, and call any part of the macroprocess *a process*. A process may include or be *a cycle*.

In Ref. [5], a controlled interaction (CI) is defined as a cycle running through the CS, the Act., the environment $\mathfrak{W}$, and the SU. To a CI cycle there correspond morphisms (Fig. 1).

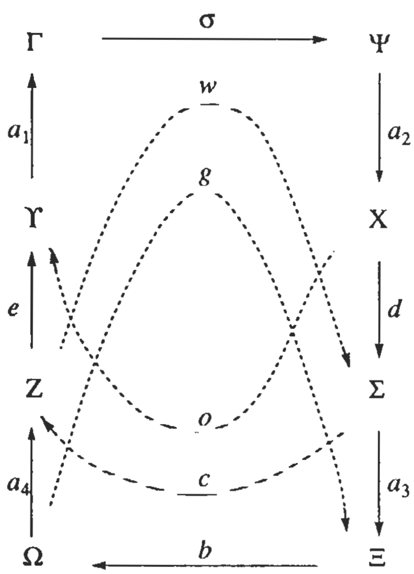

Fig. 1.

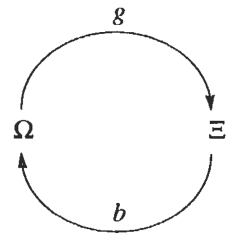

Fig. 2.

If a process contains a number n of CI cycles, it will be described by the product of relations

$$\nu_{\xi_1} b a_4 e a_1 \sigma a_2 \chi_{\iota_1} (os)^{n-1} \chi_{\iota_2} d a_3 \nu_{\xi_2}, \tag{1}$$

which must provide a mapping onto itself for each set.

Because all of the "control function" in a system is concentrated in operator b, we turn to the representation of the system as a pair of macroobjects, $\mathfrak{W}$ and $\mathfrak{B}$, to morphisms (Fig. 2), and to the product of relations $\nu_{\xi_1}(bg)^n \nu_{\xi_2}$ identical to Eq. (1), where $g : \Omega \to \Xi$.

1.3. Reflection. Pattern

Definition 1. If $\mathfrak{U}$ and $\mathfrak{R}$ each has a pair of elements s_1 and s_2, respectively, such that s_1 is a necessary condition for s_2 to exist, then s_2 is a *reflection* of element s_1 onto $\mathfrak{R}$.

Definition 2. If $\mathfrak{U}$ and $\mathfrak{B}$ each has a pair of elements s_1 and s_2, respectively, such that s_1 is a necessary condition for s_2 to occur, and satisfying a certain connection index in $\mathfrak{B}$ is a sufficient condition, then s_2 is a *pattern* of element s_1 in $\mathfrak{B}$, and s_1 is the *prepattern* of element s_2 in $\mathfrak{U}$. A pattern is the form of existence for a reflection in $\mathfrak{B}$.

We call the finite, constant-power set of specialized elements in $\mathfrak{B}$ intended to store information the memory $\mathfrak{M}$ of the CS. A memory element is characterized by its state which can vary with time.

The *set of memory* elements that could be identified with the patterns

$$\mathfrak{P}\mathfrak{D}(O_1, O_2, \ldots, O_\omega, \ldots, O_K), \quad (2)$$

where O_ω is the designation (which we call a *pseudo-identifier* or *an indicator*) of the ω-th pattern and J is a constant is called the memory of patterns we call the set the *memory of formed patterns* $\mathfrak{P}\mathfrak{D} = \{O_\omega\} \subseteq \mathfrak{M}\mathfrak{D}$ and $|\mathfrak{P}\mathfrak{D}| = k - \mathrm{var}(t)$ *conceived in CS.*

1.4. The Statement of the PFR Problem

Let there be specified $\mathbf{X}_P$, $\mathcal{H}$ and $\mathfrak{M}$, and let the law g be unknown for t_1. However, the CS establishes a CI in the manner described in Refs. [6, 7] and maintains it for an unspecified length of time. Consider the possibility, under the circumstances, of organizing in the CS an apparatus which could solve the PFR problem automatically. In seeking a solution, we use the approach outlined in Ref. [6], and in reasoning we adhere to the principles laid down in Ref. [7].

2. CONTENT AND FORM OF INPUT INFORMATION

For the CS, the source of information about system $\mathfrak{U}$ (*input information*) is the set $\mathbf{X}_P$.

2.1. Content of Input Information

The content of input information is determined by the set of macroprocesses in system $\mathfrak{U}$ incidental to the input poles $\mathbf{X}_P$. We partition the macroprocesses into processes, and the latter into routes joining macroobjects (Fig. 3). Now the content of input information is determined by the infinite number of all chain-linked routes terminating at $\mathbf{X}_P$.

We denote the set of system elements corresponding to the end of a route by the symbol "h." (for "head"), and the set of system elements corresponding to its start by the symbol "t." (for "tail").

Then for $\mathbf{X}_P$ we immediately put $\{h.dx\} = \tilde{X}''_P$

$$(\{h.yx'\} \cup \{h.\mathfrak{R}x''\}) \subset \{h.dx\} \quad (3)$$

and $\{h.yx'\} \cap \{h.\mathfrak{R}x''\} \geq \Lambda$. Also, for $\mathbf{X}_M$ we indicate that

$$\tilde{X}_M = \{h.\mathfrak{S}x\} \cup \{h.yx\},\ \{h.\mathfrak{S}x\} \cap \{h.yx\} \geq \Lambda,$$

$$\tilde{X}''_M = \{t.xy''\} \cup \{t.x\mathfrak{R}''\} \cup \{t.dx\},$$

$$(\{t.xy''\} \cup \{t.x\mathfrak{R}''\}) \cap \{t.dx\} \geq \Lambda,$$

$$\{t.xy''\} \cap \{t.x\mathfrak{R}''\} \geq \Lambda.$$

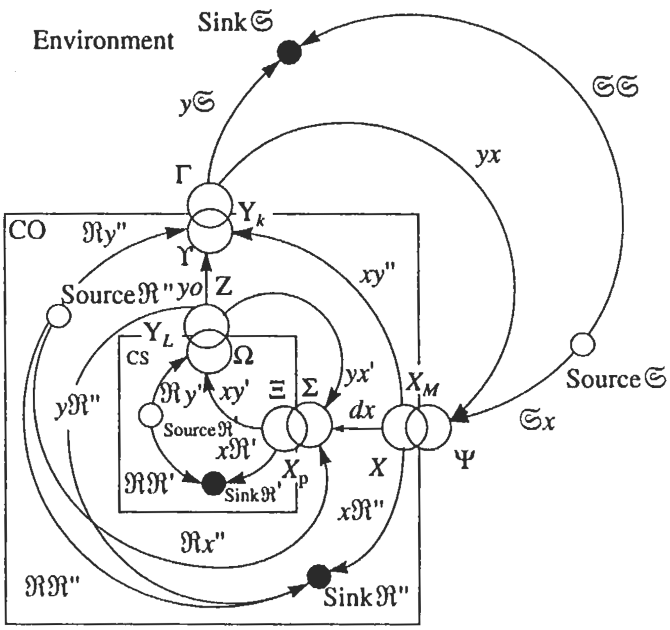

Fig. 3.

It is necessary to consider the relations $\tilde{X}_M \overset{\alpha_1}{\rightarrow} \mathbf{X}_M$ and $\mathbf{X}_M \overset{\alpha_2}{\rightarrow} \tilde{X}''_M$. We assume that $\mathbf{X}_M \cap \mathbf{X}_P = \Lambda$. Note that in the CS

$$\tilde{X}'_P = \{t.xy'\} \cup \{t.x\mathfrak{R}'\}, \quad (4)$$

$$\{t.xy'\} \cap \{t.x\mathfrak{R}'\} \geq \Lambda$$

and take into account the relations $\tilde{X}''_P \overset{\alpha_3}{\rightarrow} \mathbf{X}_P$ and $\mathbf{X}_P \overset{\alpha_4}{\rightarrow} \tilde{X}'_P$. Note also that $\tilde{X}''_M \cap \tilde{X}''_P \geq \Lambda$, $\tilde{X}_M \cap \tilde{X}'_P = \Lambda$ and $(\mathbf{X}_P \cap \mathbf{X}_L) \cup (\mathbf{X}_P \cap \mathbf{X}_M) = \Lambda$.

As is shown in Ref. [5], there is a nonzero probability for any macroprocess to be influenced by each of the remaining macroprocesses in the system, including $\mathfrak{S}\mathfrak{S}$, $\mathfrak{R}\mathfrak{R}'$, and $\mathfrak{R}\mathfrak{R}''$.

The elements $\tilde{X}'_P$ are incidental to $\mathbf{X}_P$ and $\tilde{X}''_P$ and, in consequence, to the ends of all processes terminating at $\tilde{X}''_P$. The DFs Ξ are the attributes of $\mathbf{X}_P$ and, by the same token, of the elements $\tilde{X}'_P$. In effect, the set Ξ consists of informational elements belonging to the CS and dependent on input information. Hence and from Definition 1 it follows that the DFs Ξ are reflections of the environment $\mathfrak{U}$ on the CS and that any macroprocess can influence input information. Therefore, any element of the environment $\mathfrak{U}$ can be the prepattern of a pattern.

According to Ref. [5], all system processes sensed by the CS are quasi-deterministic if the system has at least one source.

It is likewise to be realized that the presence of sinks and sources in the system $\mathfrak{U}$ is solely a consequence of the constraints imposed on the manner in which the elements and connections of the system $\mathfrak{U}$ are represented.

In summary, the content of input information may be a quasi-deterministic reflection of any processes taking place in the environment $\mathfrak{U}$.

2.2. Localization of the PFR Apparatus

We now define where the PFR apparatus is localized in the system $\mathfrak{U}$. To begin with, it takes part in control, therefore it belongs to the CI. A pattern is an informational object, therefore the PFR apparatus is located inside the CS and contains the routes xy'. Because, logically, the PFR task precedes the decision making (action generation) task likewise tackled by the CS, it follows that the PFR apparatus is located closer to the starts of the routes xy' and the decision making apparatus is located closer to their terminal points (an intersection is quite likely to take place). Obviously, the input poles $\mathbf{X}_P$, as sources of input information, belong to the PFR apparatus, and the output poles, to the decision making apparatus. Since $\mathbf{X}_P$ are incidental to $x\mathfrak{N}'$, Eq. (4), it follows that at least some of the routes $x\mathfrak{N}'$ are associated with the PFR apparatus. Some of the routes $\mathfrak{N}y'$ and $\mathfrak{N}\mathfrak{N}'$ may likewise belong to the PFR apparatus as, say, causes of interference and information leaks. Nor should we exclude the possibility of any other links existing between $\mathfrak{W}$ and $\mathfrak{B}$ in addition to $\mathbf{X}_P$ and $\mathbf{Y}_L$, regardless of the level chosen to depict the system.

2.3. The Form of Input Information

Input information owes some of its constraints and distortions to its form. In turn, the form of input information is determined by the following CS parameters: $\mathbf{X}_P$, P, α_4, Ξ, β, and Eq. (3).

The physical nature of the sensors $\mathbf{X}_P$ and of the DFs Ξ has a decisive bearing on the filter, the environmental effects it lets reach the CS, the degree to which they are mediated, and their modulation.

Eq. (3) specifies what contributions $\mathfrak{S}$, $\mathfrak{N}$, and $\mathfrak{B}$ make to the content of input information.

It is fundamentally important that the sets $\mathbf{X}_P$ and Ξ are finite. Because $|\Xi| < |V \cup E|$, this inevitably imposes constraints on the content of input information and dictates the principle of operation for the CS [6, 7]. The relation α_4 defines the manner in which input information is quantified.

3. THE PATTERN FORMATION AND RECOGNITION PRINCIPLE

3.1. Justification of the PFR Principle

We set out to justify the PFR principle expounded in Ref. [6]. As follows from the preceding, every DF ν_ξ can equiprobably be the reflection of an element in $\mathfrak{U}$, and any element in $\mathfrak{U}$ can be a prepattern. We assign to each ν_ξ one information bit $\mathfrak{b}_\xi^t \in (0, 1)$ (or a logical event $\mathfrak{b}_\xi^t \in$ (FALSE, TRUE). Further, for $t \in T$, we let $\mathfrak{b}_\xi^t = 1$ (or $\mathfrak{b}_\xi^t =$ TRUE) if ν_ξ is realized on $\mathbf{X}_P$, and $\mathfrak{b}_\xi^t = 0$ (or $\mathfrak{b}_\xi^t =$ FALSE) if it is not realized.

The set $\{\mathfrak{b}_\xi\}$, $\xi = 1, 2, \ldots, N$, contains all possible outcomes from an experiment of interaction between the CS and $\mathfrak{W}$ at time t. If this experiment yields no information for the CS, its entropy $\sum_{\xi=1}^{N} p_\xi \log_2(1/p_\xi)$ will take a maximum value, $\log_2 N$. But then all outcomes $\mathfrak{b}_\xi$ from the experiment will have equal probabilities p_ξ of occurring. If, on the other hand, the experiment does yield some useful information for the CS, then $H < \log_2 N$. This can, however, happen only if the outcomes of the experiment are not equiprobable. Because the content of the input information perceived by the CS is a reflection of $\mathfrak{U}$ in the form of a prepattern, the prepattern is then a non-equiprobable outcome $\mathfrak{b}_\xi$ of the experiment at time t for the CS. Since we have ruled out *a priori* information about $\mathfrak{U}$, it remains to be supposed that the unequal probabilities of the outcomes $\mathfrak{b}_\xi$ expected by the CS at time t are unequal empirical frequencies $\mathfrak{n}_\xi/k$ of outcomes ν_ξ on a sample of $k = |(t_1, t_2, \ldots, t_p)|$ experiments.

Assuming that the logical events $\mathfrak{b}_\xi$ are nonmodal and that the probabilities

$$p_\xi \approx \mathfrak{n}_\xi / k \tag{5}$$

are discrete, we find that the CS will perceive a concrete ν_ξ as the reflection of a prepattern in $\mathfrak{U}$ only when p_ξ exceeds some threshold value

$$\mathfrak{n}_\xi / k \geq \mathfrak{m}_\omega / k \tag{6}$$

(the meaning of the subscript ω will be explained in Eq. (9)).

By choosing a statistically acceptable value for p_ξ, we can find the respective number $\mathfrak{m}_\omega \approx kp_\xi$, where k, $1 \leq k \leq |(t_1, \ldots, t_f)|$, is found by trial and error. On eliminating k from Eq. (6), we conclude that the CS must sense every ν_ξ as a prepattern if the number $\mathfrak{n}_\xi$ of events $\mathfrak{b}_\xi =$ TRUE exceeds the specified fixed value $\mathfrak{m}_\omega$. The condition

$$\mathfrak{n}_\xi = \mathfrak{m}_\omega \tag{7}$$

is a necessary condition for forming a pattern for which the prepattern in Ξ is ν_ξ.

Let us call ν_ξ the *actual* prepattern. It is a reflection of the *true* prepattern distributed in the environment $\mathfrak{U}$.

The instant $t_\omega^{fo} \in T$ at which Eq. (7) is satisfied is called the *instant of pattern formation* for the prepattern ν_ξ. There may be no t_ω^{fo} for the pattern with the subscript ω.

The instant t_ω^{ro} when b_ξ^t = TRUE and

$$\mathcal{n}_\xi > \mathcal{m}_\omega \quad (8)$$

is the *instant* at which the pattern of the prepattern ν_ξ *is recognized*. If $\mathcal{n}_\xi > \mathcal{m}_\omega$ and b_ξ^t = FALSE, then the pattern of the prepattern ν_ξ has been formed but has yet to be recognized.

We now turn to sufficient conditions for the pattern of the prepattern ν_ξ to be formed and recognized.

Obviously, the formed pattern O_ω of a prepattern ν_ξ must belong to $\mathfrak{PO}$ and there must exist the mapping

$$\text{fo} : \Xi \to \mathfrak{M} \text{ or } \nu_\xi \text{fo} O_\omega. \quad (9)$$

We denote by O_ω^t = TRUE the event which consists in that the pattern O_ω is recognized at time t; otherwise, we denote it by O_ω^t = FALSE. We introduce a condition ℓ_ω such that ℓ_ω = TRUE if the image $O_\omega = \nu_\xi \text{fo}$ has been recognized, that is, if $\mathcal{n}_\xi \geq \mathcal{m}_\omega$, and ℓ_ω^t = FALSE, otherwise.

Then the relation fo can be expressed in terms of two functions. One is the conjunction

$$O_\omega^{t+1} = b_\xi^t \,\&\, \ell_\omega^t, \quad O_\omega^t \in (\text{TRUE, FALSE}). \quad (10)$$

The other, f, establishes the taxonomic relation between ξ and ω

$$\omega = \xi f \text{ or } \omega = f(\xi), \quad (11)$$

then $J \leq N$.

The prepattern in Ξ of the pattern O_ω is $\nu_\xi \in \{\nu_\xi\}$, where

$$\{\xi\} = \omega f^{-1}. \quad (12)$$

In view of Eq. (12), we interpret Eq. (10) as $O_\omega^{t+1} = (\bigvee_{\xi \in \omega f^{-1}} b_\xi^t) \,\&\, \ell_\omega^t$ or, on setting $b_\omega^t = \bigvee_{\xi \in \omega f^{-1}} b_\xi^t$, as $O_\omega^{t+1} = b_\omega^t \,\&\, \ell_\omega^t$, and instead of $\mathcal{n}_\xi$ in Eqs. (7, 8) we use

$$\mathcal{n}_\omega = \sum_{\xi \in \omega f^{-1}} \mathcal{n}_\xi. \quad (13)$$

Note also that the operating principle of the CS [6, 7] can be interpreted as a purposeful increase by the control system of the empirical frequencies $\mathcal{n}_\xi / k$ of the realizations of DFs ν_ξ meeting certain criteria.

3.2. Parameter Constraints Imposed by the PFR Principle

Let f be a one-to-one mapping. Then ω is the address of the *memory* location s_ω *of the patterns* $\mathfrak{MO} \subset \mathfrak{M}$ *conceivable in the CS*, whereas $\mathfrak{PO} \subseteq \mathfrak{MO}$, and

$$s_\omega \begin{cases} \in \mathfrak{PO} \text{ when } \ell_\omega = \text{TRUE}, \\ \in \mathfrak{MO} \setminus \mathfrak{PO} \text{ when } \ell_\omega = \text{FALSE}. \end{cases}$$

Let there be specified all $\mathcal{m}_\omega$, $\omega = 1, 2, \ldots, N$; $N = 2^P$. We take a number N of r_ω-bit locations s_ω to store binary numbers, where

$$r_\omega = [\log_2 \mathcal{m}_\omega + 1] \quad (14)$$

and a 0 (FALSE) or a 1 (TRUE) can be written into each bit. Then the size of the memory $\mathfrak{MO}$ is

$$\sum_{\omega=1}^{N} r_\omega = \sum_{\omega=1}^{2^P} \left[\log_2 \mathcal{m}_\omega + 1\right] \text{ bits.}$$

At each time t, a binary number $\mathcal{n}_\omega^t$ is written into each location s_ω by the following rule:

$$\begin{cases} \mathcal{n}_\omega^1 = 0; \\ \mathcal{n}_\omega^{t+1} = \begin{cases} \mathcal{n}_\omega^t + b_\omega^{t+1} \text{ when } \ell_\omega^t = \text{FALSE}, \\ \mathcal{m}_\omega \text{ when } \ell_\omega^t = \text{TRUE}; \end{cases} \\ \ell_\omega^t = \begin{cases} \text{FALSE when } \mathcal{n}_\omega^t < \mathcal{m}_\omega, \\ \text{TRUE when } \mathcal{n}_\omega^t \geq \mathcal{m}_\omega; \end{cases} \\ t_\omega^{fo} = t \text{ when } \mathcal{n}_\omega^t = \mathcal{m}_\omega. \end{cases} \quad (15)$$

Then to the set of formed patterns $\mathfrak{PO} = \{O_\omega\}$ there corresponds the set $\{s_\omega \mid \ell_\omega^t = \text{TRUE}\}$.

Because the locations s_ω are identified by a mapping f (in the general case, by a mapping $\{\xi\}$ onto $\{\omega\}$ and not necessarily a one-to-one mapping), there is no need to expend any of the memory $\mathfrak{M}$ to hold the addresses of the locations s_ω. Information retrieval from s_ω likewise uses a fixed mapping [6, 7]. Recall also that the CS can access s_ω only in the course of interaction with $\mathfrak{W}$ via the cycles $yo \to yx \to dx \to xy'$ or $yx' \to xy'$.

We now consider the case where the empirical frequencies $c(\xi) = \mathcal{n}_\xi / k$ are small and comparable with

$1/N$. That is, $n'_\xi < m_\omega$ everywhere on T for the specified m_ω and T. Then by the prepatterns in Ξ we mean certain subsets $\{\kappa^\Xi_{z,\xi}\}$ of sets ν_ξ. We set $\gamma_{i,\xi} = \{\kappa^\Xi_{z,\xi}\} \subset \nu_\xi$, $m_{i,\xi} = |\gamma_{i,\xi}|$. Let $\bigcup_{\forall i} \gamma_{i,\xi} \subseteq \nu_\xi$, and $\bigcap_{\forall i} \gamma_{i,\xi} \geq \Lambda$. We further assume that

$$\gamma_{i_1,\xi_1} = \gamma_{i_2,\xi_2} \tag{16}$$

leads to $i_1 = i_2$ and $\xi_1 = \xi_2$. Then $c(i, \xi) = n_{i,\xi}/k$ is the empirical frequency of the event $b'_{i,\xi}$ = TRUE on the sample T, $k = |T|$ and $\sum_{\xi=1}^{N}\sum_{i=1}^{N'} c(i, \xi) = 1$. Let there be specified the numbers $m_{i,\omega}$. Then by decreasing the value of $m_{i,\xi}$ from $m_{i,\xi} = \mathrm{P} = |\nu_\xi|$ to $m_{i,\xi} = 1$, we find $m_{i,\xi}$ such that $n_{i,\xi} > m_{i,\omega}$.

If no such $m_{i,\xi}$ are found, then, as prepatterns in the system in question, we should take the events $b_{i,\xi}$ = TRUE, which are even less probable than $p_{i,\xi} = m_{i,\xi}/k$, and decrease the numbers $m_{i,\omega}$ as far as $m_{i,\omega} = 1$.

Suppose we have found certain $m_{i,\xi} > 1$ and $m_{i,\omega} > 1$ such that it is legitimate to speak of prepatterns in Ξ. Then we should have to establish, as in the case of f, the mapping of $\{(i, \xi)\}$ onto $\{\omega\}$. Note, however, that $\gamma_{i,\xi} \subset \nu_\xi$ is included in a set $\{\nu_{\xi'}\}$ rather than in a single ν_ξ, and that $\xi \in \{\xi'\}$. The DFs common to $\nu_{\xi'}$ are $\kappa^\Xi_{z,\xi'} = \kappa^\Xi_{z,\xi} \in \{\kappa^\Xi_z\} | \kappa^\Xi_{z,\xi} \in \gamma_{i,\xi}$, and the remaining z's are represented by all possible combinations of κ^Ξ_z from the sets $\{\kappa^\Xi_z\} | \kappa^\Xi_{z,\xi} \notin \gamma_{i,\xi}$.

We limit ourselves to the case $\{\kappa^\Xi_z\} = (0, 1)$, $z = \overline{1, \mathrm{P}}$. We interpret $\kappa^\Xi_z = 1$ as the operation of a sensor x_z on satisfying the condition β, whereas $\kappa^\Xi_z = 0$, indicates that the sensor has not operated. We take it that a valid signal is a 1 and not a 0. Then

$$\kappa^\Xi_{z,\xi} = 1 | \kappa^\Xi_{z,\xi} \in \gamma_{i,\xi}. \tag{17}$$

Because all $\kappa^\Xi_{z,\xi}$ lying in the domain of the function f are 1's, then f is defined on the subscripts z and ξ, and not on the values of $\kappa^\Xi_{z,\xi}$.

Let there be specified $\gamma_{i,\xi}$. We will write κ_z instead of $\kappa^\Xi_z = 1$ if, for the specified z, $\kappa^\Xi_{z,\xi} \in \gamma_{i,\xi}$, and υ_z instead of $\kappa^\Xi_z = 1$ or $\bar{\upsilon}_z$ instead of $\kappa^\Xi_z = 0$ if, for the specified z, $\kappa^\Xi_{z,\xi} \notin \gamma_{i,\xi}$. Then it will be an easy matter to enumerate all ν_ξ's which include $\gamma_{i,\xi}$. They are

$$(\upsilon_1 \cup \bar{\upsilon}_1 \cup \kappa_1, \upsilon_2 \cup \bar{\upsilon}_2 \cup \kappa_2, \ldots, \upsilon_z \cup \bar{\upsilon}_z \cup \kappa_z, \ldots, \upsilon_\mathrm{P} \cup \bar{\upsilon}_\mathrm{P} \cup \kappa_\mathrm{P}), \tag{18}$$

namely

$$(\upsilon_1 \cup \kappa_1, \upsilon_2 \cup \kappa_2, \ldots, \upsilon_\mathrm{P} \cup \kappa_\mathrm{P}),$$
$$(\upsilon_1 \cup \kappa_1, \upsilon_2 \cup \kappa_2, \ldots, \bar{\upsilon}_\mathrm{P} \cup \kappa_\mathrm{P}),$$
$$\ldots$$
$$(\bar{\upsilon}_1 \cup \kappa_1, \bar{\upsilon}_2 \cup \kappa_2, \ldots, \bar{\upsilon}_\mathrm{P} \cup \kappa_\mathrm{P}).$$

It is not convenient to identify $\gamma_{i,\xi}$ with the pair of subscripts (i, ξ). Instead, we will use the inverse function $\gamma_\omega = \omega f^{-1}$ for the purpose, where

$$\gamma_\omega = \{\kappa^\Xi_{z,\xi}\} \subseteq \bigcap_{\forall \xi'} \nu_{\xi'}, \tag{19}$$

and $\{\nu_{\xi'}\}$ is found from Eq. (18). Then f is a mapping of $\{\xi\}$ into $\{\omega\}$.

The relations $|\{\nu_{\xi'}\}| = 2^{\mathrm{P}-m_\omega}$, $c(\xi') = \sum_{\forall \xi'} n_{\xi'}/k \sim |\{\nu_{\xi'}\}|$ and $p(\xi') = \sum_{\forall \xi'} p(\xi) \sim |\{\nu_{\xi'}\}|$, where $m_\omega = |\gamma_\omega|$ imply that, in the case of equal *a priori* probabilities $p(\xi)$, Eq. (6), decreasing m_ω by a factor of n will increase the probability $p(\xi')$ of the prepattern γ_ω by a factor of $2^{m_\omega(1-1/n)}$. Thus, changing from f as a one-to-one mapping of $\{\xi\}$ onto $\{\omega\}$ to f as a mapping of $\{\xi\}$ onto $\{\omega\}$ offers an efficient way to increase the empirical frequencies of prepatterns and, thus, to enhance the probability of pattern formation in the CS.

Definition 3. We say that the pattern O_a more *adequately fits* the actual prepattern than does the pattern O_b if the value $2^{\mathrm{P}-m_a}$ corresponding to O_a is smaller than the value $2^{\mathrm{P}-m_b}$ corresponding to O_b.

In the general case, the powers m_ω may be distributed anywhere from P to 1. For the specified $\mathbf{X}_\mathrm{P}$ and Ξ, one may rate as optimal a partition of Ξ into γ_ω and a distribution $m_\omega(\omega)$ such as would ensure a maximum number of formed patterns essential for control purposes. In a nonoptimal case, a need will arise for the statistical redundancy of the sets $\{\gamma_\omega\}$.

3.3. Pattern Indication

According to the implication of the signal O'_ω = TRUE, its time extent should be defined by the interval $[t^{ro}_\omega, t^s_\omega]$, where t^{ro}_ω represents the instant when the event b'_ω = TRUE occurs, and t^s_ω represents the instant of arrival of the signal S'_ω = TRUE which does not